Metaheuristic Optimization: Differential Evolution (DE)

Adaptive and Cooperative Algorithms (ECE 457A)

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Differential Evolution: the Idea

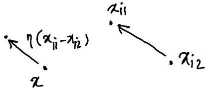
- Differential Evolution (DE) is one of the evolutionary algorithms. It was proposed in 1996-1997 [1, 2].
- Recall genetic algorithm which has both crossover and mutation where crossover is performed and then mutation is done.
- DE performs **mutation and then crossover**.
- Its **mutation** is also in a specific form which we will discuss.
- Initially, the candidate solutions are **randomly initialized** in the optimization landscape.
- Optional (we can select all of them): In later iterations, some of the best candidate solutions are selected by **natural selection**. We can use any natural selection technique some of which were discussed for genetic algorithm.
- Then, **mutation** is performed.
- Afterwards, **crossover** is performed where any of the crossover methods, introduced for genetic algorithm, can be used.
    - If both parent are mutated, the offspring might become too much off.
    - If none of parents are mutated, then why did we mutate in the first place?
    - Therefore, for every crossover, **one of the parents is mutated and the other is not mutated**.

Differential Evolution: Mutation

- In DE, the **mutation** of a candidate solution $\boldsymbol{x}_i$ is usually in the following form:

$$\boldsymbol{x}_i := \boldsymbol{x} + \eta(\boldsymbol{x}_{i1} - \boldsymbol{x}_{i2}), \tag{1}$$

  where $\boldsymbol{x}$ is the starting point for mutation, $\eta \in \mathbb{R}$ is the **scaling factor** and $\boldsymbol{x}_{i1}$ and $\boldsymbol{x}_{i2}$ are either random points, corresponding to $\boldsymbol{x}_i$, or two best solutions so far.



- Later attempts of DE used a set of paired points per mutation of a candidate solution:

$$\boldsymbol{x}_i := \boldsymbol{x} + \eta \sum_{j=1}^{p} (\boldsymbol{x}_{ij1} - \boldsymbol{x}_{ij2}), \tag{2}$$

  where p is the number of pairs per mutation and $(\boldsymbol{x}_{ij1}, \boldsymbol{x}_{ij2})$ is the j-th pair corresponding to $\boldsymbol{x}_i$. These pairs can be either random points or the best solutions so far.

- The larger the scaling factor η is, the more **exploration** and the less **exploitation** we have. Therefore, the scaling factor can be larger in the initial iterations and it can be decremented gradually so we have more exploration initially and more exploitation later in the iterations.

Variants of Differential Evolution

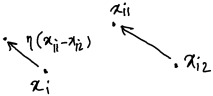
- DE has various variants where the variants can be denoted by:

$$x/y/z, \tag{3}$$

  where x stands for the starting point of the mutation, y is for the number of pairs per mutation, and z is for the crossover method.

- Some of the most well-known variants of DE are:
  - $\boldsymbol{x}_i/1/z$: starting from the current solution, a pair (either random or best solutions so far), any crossover method.

$$\boldsymbol{x}_i := \boldsymbol{x}_i + \eta(\boldsymbol{x}_{i1} - \boldsymbol{x}_{i2}). \tag{4}$$



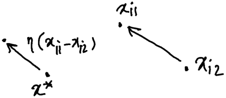
  - $\boldsymbol{x}_i/p/z$: starting from the current solution, p pairs (either random or best solutions so far), any crossover method.

$$\boldsymbol{x}_i := \boldsymbol{x}_i + \eta \sum_{j=1}^{p} (\boldsymbol{x}_{ij1} - \boldsymbol{x}_{ij2}). \tag{5}$$

Variants of Differential Evolution

- Some of the most well-known variants of DE are:
  - $\boldsymbol{x}^*/1/z$: starting from the best solution so far, a pair (either random or best solutions so far), any crossover method.

$$\boldsymbol{x}_i := \boldsymbol{x}^* + \eta(\boldsymbol{x}_{i1} - \boldsymbol{x}_{i2}). \tag{6}$$



  - $\boldsymbol{x}^*/p/z$: starting from the best solution so far, p pairs (either random or best solutions so far), any crossover method.

$$\boldsymbol{x}_i := \boldsymbol{x}^* + \eta\sum_{j=1}^{p}(\boldsymbol{x}_{ij1} - \boldsymbol{x}_{ij2}). \tag{7}$$

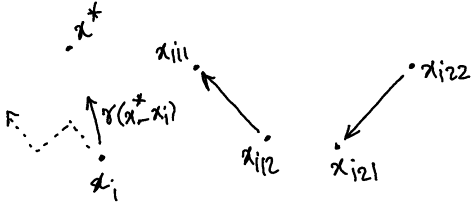
Variants of Differential Evolution

- Some of the most well-known variants of DE are:
  - current-to-best/p/z: starting from a random point between the current solution and the best solution so far, p pairs (either random or best solutions so far), any crossover method.

$$\boldsymbol{x}_i := \gamma \boldsymbol{x}^* + (1-\gamma)\boldsymbol{x}_i + \eta \sum_{j=1}^{p} (\boldsymbol{x}_{ij1} - \boldsymbol{x}_{ij2}), \text{ or} \tag{8}$$

$$\boldsymbol{x}_i := \boldsymbol{x}_i + \gamma(\boldsymbol{x}^* - \boldsymbol{x}_i) + \eta \sum_{j=1}^{p} (\boldsymbol{x}_{ij1} - \boldsymbol{x}_{ij2}), \tag{9}$$

where $\gamma \in [0, 1]$.



Differential Evolution: Algorithm

Algorithm Differential Evolution

Initialize the candidate solutions $\{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\}$
while *not converged* **do**
  for *each candidate solution* $\boldsymbol{x}_i \in \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\}$ **do**
    Draw p random points in the feasibility set
    $\boldsymbol{x}_i := \boldsymbol{x}^* + \eta \sum_{j=1}^{p} (\boldsymbol{x}_{ij1} - \boldsymbol{x}_{ij2})$
    if *better cost* **then**
      Update the solution
    Perform crossover for several (mutated, non-mutated) parent pairs
    $\{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\} \leftarrow$ Perform natural selection
Return the solution $\boldsymbol{x}$

Acknowledgment

- Some slides of this slide deck are inspired by teachings of Prof. Saeed Sharifian at the Amirkabir University of Technology, Department of Electrical Engineering.
- Some books on DE: (2006, 2013) [3, 4]
- Some surveys on DE: (2010, 2016, 2019, 2020) [5, 6, 7, 8]

References

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