Game Theory:
Dynamic Games

Adaptive and Cooperative Algorithms (ECE 457A)

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Dynamic Games

Dynamic Games

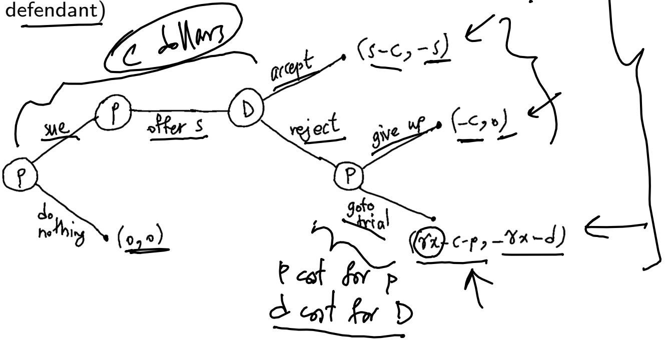
- When the game has some ranges and parameters, the game becomes **dynamic**.
- In dynamic games, we can find the ranges of parameters which are suitable for the players' payoffs.

Example 1

Example 1

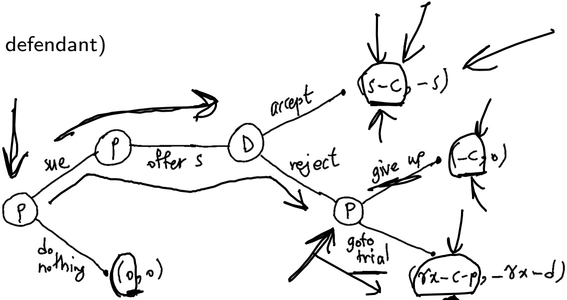
$$E(X) = \sum_{x} x \underbrace{P(X)}_{\gamma}$$

- Players: plaintiff and defendant
- The order of playing game:
  - The plaintiff decides whether to bring suit against the defendant at cost c.
  - The plaintiff makes a take-it-or-leave-it settlement offer of $s > 0$.
  - The defendant accepts or rejects the settlement offer.
  - If the defendant rejects the offer, the plaintiff decides whether to give up or go to trial at a cost p (cost of lawyer) to itself and cost d to the defendant.
  - If the case goes to trial, the plaintiff wins amount x with probability γ and otherwise wins nothing.
- Payoffs: (plaintiff, defendant)



Example 1

- Payoffs: (plaintiff, defendant)



- The plaintiff sues if:

$$s - c > 0 \implies s > c, \tag{1}$$

$$\gamma x - c - p > 0 \implies \gamma x - p > c. \tag{2}$$

- In case the plaintiff sues and the defendant rejects the settlement, the plaintiff will go to trial if:

$$\gamma x - \cancel{c} - p > -\cancel{c} \implies \gamma x > p. \tag{3}$$

- The plaintiff prefers the settlement to not suing at all:

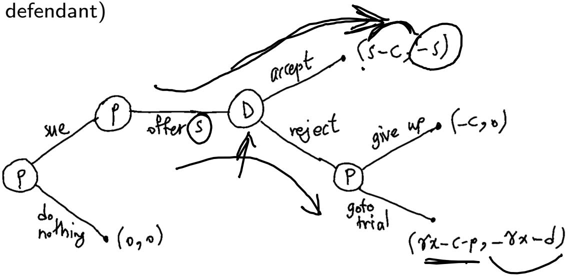
$$s - c > 0 \implies s > c. \tag{4}$$

- The plaintiff prefers the settlement to trying again:

$$s - c > \gamma x - c - p \implies s > \gamma x - p. \tag{5}$$

Example 1

- Payoffs: (plaintiff, defendant)



- **Bargaining**:
  - The plaintiff prefers settlement if:

$$s - c > \gamma x - c - p \implies \boxed{s > \gamma x - p.} \tag{6}$$

  - The defendant prefers settlement if:

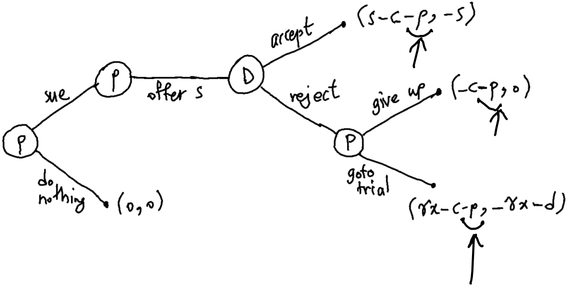
$$-s > -\gamma x - d \implies \boxed{s < \gamma x + d.} \tag{7}$$

  - Equilibrium: $\boxed{s = \gamma x + d.}$

Example 2

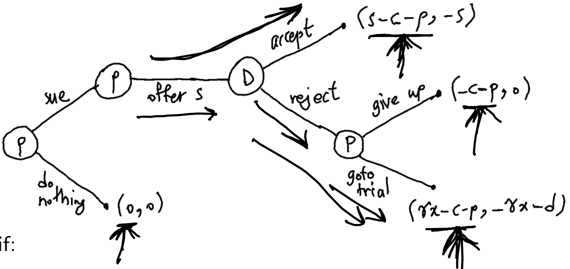
Example 2

- Difference from the previous game: the plaintiff pays in advance, without any refund if the case settles.
- Payoffs: (plaintiff, defendant)



Example 2

- Payoffs: (plaintiff, defendant)



- The plaintiff sues if:

$$s - c - p > 0 \implies s > c + p, \tag{8}$$
$$\gamma x - c - p > 0 \implies \gamma x - p > c. \tag{9}$$

- In case the plaintiff sues and the defendant rejects the settlement, the plaintiff will go to trial if:

$$\gamma x - c - p > -c - p \implies \gamma x > 0. \tag{10}$$

- The plaintiff prefers the settlement to not suing at all:

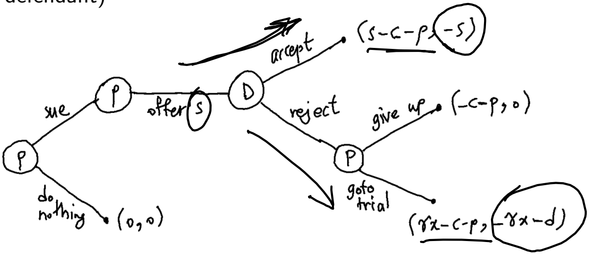
$$s - c - p > 0 \implies s > c + p. \tag{11}$$

- The plaintiff prefers the settlement to trying again:

$$s - c - p > \gamma x - c - p \implies s > \gamma x. \tag{12}$$

Example 2

- Difference from the previous game: the plaintiff pays in advance, without any refund if the case settles.
- Payoffs: (plaintiff, defendant)



- **Bargaining**:
  - The plaintiff prefers settlement if:

$$s - c - p > \gamma x - c - p \implies \boxed{s > \gamma x.} \tag{13}$$

  - The defendant prefers settlement if:

$$-s > -\gamma x - d \implies \boxed{s < \gamma x + d.} \tag{14}$$

  - So, the **settlement range** is $s \in (\gamma x, \gamma x + d)$. But as the plaintiff offers the settlement, the **equilibrium** is $s = \gamma x + d$ because the plaintiff wants the largest possible s value.

Acknowledgment

- Some slides of this slide deck are inspired by teachings of Prof. Stanko Dimitrov at the University of Waterloo, Department of Management Science and Engineering.
- Some slides of this slide deck are based on the following book: Eric Rasmusen, "Games and Information: An Introduction to Game Theory", 4th Edition, 2007, [1] `https://www.rasmusen.org/GI/download.htm`

References

[1] E. Rasmusen, *Games and information: An introduction to game theory*.
Wiley-Blackwell, 4 ed., 2007.