Metaheuristic Optimization: Evolutionary Programming (EP)

Adaptive and Cooperative Algorithms (ECE 457A)

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- Evolutionary Programming (EP) is one of the evolutionary algorithms. It was invented by Lawrence J. Fogel while serving at the <u>National Science Foundation</u> in <u>1960</u> [1, 2, 3].
- Recall genetic algorithm which has both crossover and mutation.
- EP only contains <u>mutation</u> and not crossover. It updates the candidate solutions by <u>mutation</u>. a specific Type of nutation
- Initially, the candidate solutions are randomly initialized in the optimization landscape.
- Optional: In later iterations, some of the best candidate solutions are selected by <u>natural</u> <u>selection</u> among the candidate solutions before and <u>after mutations</u>. We can use any natural selection technique some of which were discussed for genetic algorithm.

Evolutionary Programming: Mutation

• The update of a selected candidate solution $x_i \in \mathbb{R}^d$ after natural selection is:

$$\begin{array}{c} (\downarrow) \\ (\downarrow)$$

where η is the scaling factor (noise factor), $\phi(.)$ is the scaling function, and σ is the strategy parameter.



- Usually the change Δx_i is large in initial iterations for more <u>exploration</u> and it gets smaller gradually by iteration for having more <u>exploitation</u>.
- The larger the scaling factor η is, the more exploration and the less exploitation we have. Therefore, the scaling factor can be larger in the initial iterations and it can be decremented gradually so we have more exploration initially and more exploitation later in the iterations.

Evolutionary Programming: η

- The scaling factor (noise factor) is a stochastic variable and can be sampled from various distributions such as:
 - Uniform distribution:

$$\eta \sim \mathbb{P}(\eta) = \underbrace{U(\eta_{\min}, \eta_{\max})}_{(3)}.$$

Gaussian distribution:

$$\eta \sim \mathcal{N}(0, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\eta^2}{2\sigma^2}}.$$
(4)

Cauchy distribution:

$$\eta \sim \mathbb{P}(\eta) = \frac{1}{\pi(1+\eta^2)}.$$
(5)

Evolutionary Programming: $\phi(.)$

• The scaling function can be any of the following functions:

Static scaling function:

$$\phi(\sigma) = \alpha \sigma, \tag{6}$$

where α is a constant scale.

- Dynamic scaling function: for example, it can be a function of the iteration index and σ .
- Self-adaptive scaling function: according to the error, it adjusts itself. For example, depending on whether the cost is getting better or worse, it adapts.

Evolutionary Programming: σ

 In general, the strategy parameter can be a function applied on elements of each candidate solution differently. The strategy parameter for the <u>j-th dimension of the i-th</u> candidate solution can be denoted as:

$$\sigma = \sigma_{ij}(t), \tag{7}$$

where t is the iteration index (it is a function of the iteration index).

- The strategy parameter can be any of the following functions:
 - Static strategy parameter (not very good):

$$\sigma = \sigma_{ij}(t) = \sigma_{ij}.$$
(8)

Dynamic strategy parameter:

$$\sigma = \sigma_{ij}(t) \stackrel{\Psi}{=} \sigma_i(t) = f(\mathbf{x}_i), \qquad (9)$$

where x_i is the *i*-th candidate solution and $f(x_i)$ is the cost function at x_i .

- It makes sense as the update σ will be smaller if the cost is small as we need to exploit there rather than exploring.
- * Its flaw is that we are using absolute cost and not the relative cost value.
- * We can also use any increasing monotonic function of $f(x_i)$, e.g., $g(f(x_i))$.

Evolutionary Programming: σ



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- The strategy parameter can be any of the following functions:
 - Relatively dynamic strategy parameter:

$$\frac{V}{\sigma = \sigma_{ij}(t) = \sigma_i(t) = |f(\mathbf{x}_i) - f^*|,}$$
(10)

where f* is the best (smallest) cost value found so far and |.| is the absolute value.
We can also specify the direction of movement (update) to be toward the best solution

found so far. For this, we can multiply this direction by any of the above-mentioned update amounts; for example:

$$\underbrace{\sigma = \sigma(\mathbf{x}^* - \mathbf{x}_i),}_{|c(x)| = |c(x)|} \sigma_i \in (0, 1),$$
(11)

$$\sigma = \frac{|f(\mathbf{x}_{i}) - f^{*}|}{|f(\mathbf{x}_{i}) + f^{*}|} (\mathbf{x}^{*} - \mathbf{x}_{i}),$$
(12)

where x^* denotes the best candidate solution found so far and $x^* - x_i$ is a vector connecting x_i to x^* .

Evolutionary Programming: Variants

- Based on the which scaling factor, scaling function, and strategy parameter we choose, we can have various variants of evolutionary programming. Some of the well-known variants are:
 - Classical Evolutionary Programming (1960) [1, 2]
 - Fast Evolutionary Programming (1996) [4]
 - Accelerated Evolutionary Programming (1996) [5]
 - Exponential Evolutionary Programming (2005-2006) [6, 7]
 - Momentum Evolutionary Programming (adds a momentum term in the update)
 (2012) [8]

Evolutionary Programming: Algorithm



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