Metaheuristic Optimization: Particle Swarm Optimization (PSO)

Adaptive and Cooperative Algorithms (ECE 457A)

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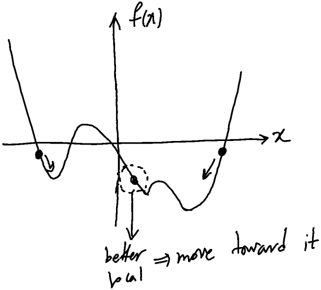
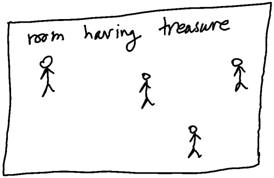
Swarm Optimization: the Idea

- Some of the metaheuristic optimization algorithms are **swarm optimization** methods.
- In swarm methods, we have a **swarm of particles** which collaboratively try to find the global best in an optimization landscape.
- The swarm methods are usually (but not always) **bio-inspired** or **nature-inspired** algorithms where the particles behave like animals, birds, creatures, etc.
- A recent **survey** on nature-inspired optimization is published in 2023 [1].
- For example, a swarm method can be inspired by a flock of birds or group of fish.
- Many bio-inspired or swarm metaheuristic algorithms exist such as:
  - **Particle Swarm Optimization (PSO)**: 1995 [2]
  - **Ant colony**: 1996 [3, 4]
  - **Grey wolf** optimizer: 2014 [5]
  - **Whale** optimization algorithm: 2016 [6]
  - **Salp** Swarm Algorithm: 2017 [7]
  - A scholar in this area: **Seyedali Mirjalili**, Torrens University Australia, Australia, `https://scholar.google.com/citations?user=TJHmrREAAAAJ&hl=en&oi=sra`

Particle Swarm Optimization: the Idea

- **Particle Swarm Optimization (PSO)** was proposed in 1995 [2].
- The idea of PSO is like finding a treasure by a group of people.





- It is inspired a flock of birds or group of fish. Hence, it can be seen as one of the **bio-inspired** metaheuristic algorithms or **swarm optimization**.

Particle Swarm Optimization: the Formula

- The candidate solutions are the **particles** (vectors).
- Every particle **searches locally** in a local neighborhood.
- Three components for the velocity vector for updating the solution:
  - the **momentum (history)** of previous velocity (fro exploitation): $\alpha_1 \boldsymbol{v}_i^{(k)}$
  - update according to the **local best** in the iteration: $\alpha_2(\boldsymbol{x}_{\text{localBest}}^{(k)} - \boldsymbol{x}_i^{(k)})$
  - update according to the **global best** in the iteration: $\alpha_3(\boldsymbol{x}_{\text{globalBest}}^{(k)} - \boldsymbol{x}_i^{(k)})$
- The update of every particle:

$$\boldsymbol{v}_i^{(k+1)} := \alpha_1 \boldsymbol{v}_i^{(k)} + \boxed{\alpha_2(\boldsymbol{x}_{\text{localBest}}^{(k)} - \boldsymbol{x}_i^{(k)})} + \boxed{\alpha_3(\boldsymbol{x}_{\text{globalBest}}^{(k)} - \boldsymbol{x}_i^{(k)})}, \tag{1}$$

$$\boldsymbol{x}_i^{(k+1)} := \boldsymbol{x}_i^{(k)} + \boldsymbol{v}_i^{(k+1)}. \tag{2}$$

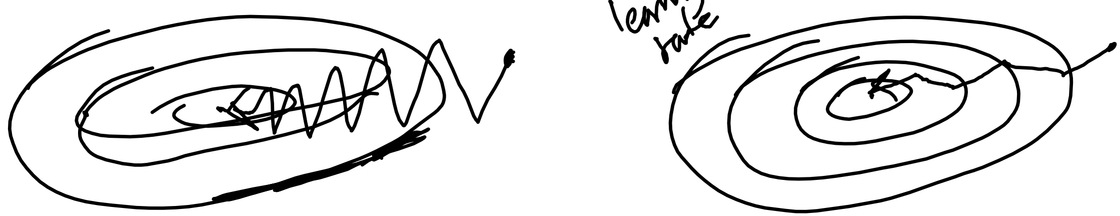
  where α_1, α_2, and α_3 are weight (regularization) hyper-parameters.
- Variants of PSO:
  - local best:
    - local best of particle itself in this iteration
    - local best of particle itself so far (in all iterations until now)
  - global best:
    - global best of all iterations so far (best found solution so far)
    - best of solutions found by particles in this iteration

$$\begin{cases} V^{(k+1)} = V^{(k)} + \Delta V^{(k+1)} \\ \Delta V^{(k+1)} = -\eta \dfrac{\partial V}{\partial W} \rightarrow \text{gradient} \end{cases}$$

$$\Delta V^{(k+1)} = -\eta \frac{\partial V}{\partial W} + \alpha \Delta V^{(k)}$$

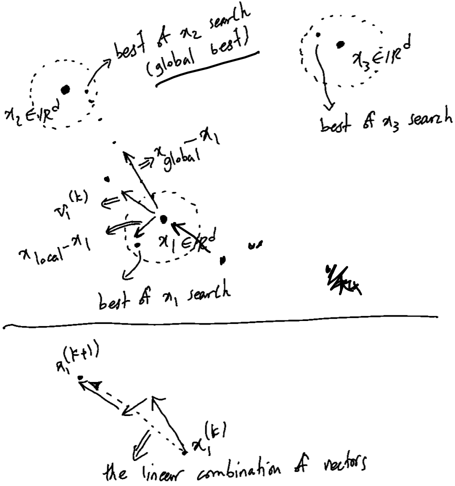
learning rate

momentum rate



Particle Swarm Optimization: Visualizing the Formula

$$
\begin{aligned}
\mathbf{v}_i^{(k+1)} &:= \alpha_1 \mathbf{v}_i^{(k)} + \alpha_2(\mathbf{x}_{\text{localBest}}^{(k)} - \mathbf{x}_i^{(k)}) + \alpha_3(\mathbf{x}_{\text{globalBest}}^{(k)} - \mathbf{x}_i^{(k)}),\\
\mathbf{x}_i^{(k+1)} &:= \mathbf{x}_i^{(k)} + \mathbf{v}_i^{(k+1)}.
\end{aligned}
$$



Particle Swarm Optimization: Flowchart

append(best csts)

argmin(csts)

min cost

x^*

if current cost < best cost

bestcost ← current cost

bestx ← x

Acknowledgment

- Some slides of this slide deck are inspired by teachings of Prof. Saeed Sharifian at the Amirkabir University of Technology, Department of Electrical Engineering.

- A good web link about PSO: https://www.analyticsvidhya.com/blog/2021/10/an-introduction-to-particle-swarm-optimization-algorithm/

References

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[2] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proceedings of ICNN'95-international conference on neural networks*, vol. 4, pp. 1942–1948, IEEE, 1995.

[3] M. Dorigo, V. Maniezzo, and A. Colorni, "Ant system: optimization by a colony of cooperating agents," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 26, no. 1, pp. 29–41, 1996.

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