

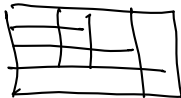
Metaheuristic Optimization: Tabu Search

Adaptive and Cooperative Algorithms (ECE 457A)

ECE, MME, and MSCI Departments,
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A hand-drawn diagram of a rectangular structure, possibly representing a book or a container. It features a vertical line on the right side and two horizontal lines on the left side, creating three horizontal compartments. The top-left corner is further divided into four small rectangular sections by vertical lines.



- Tabu search is a **meta-algorithm** which can be used with other metaheuristic optimization algorithms, such as local search or PSO.
- It was proposed in 1986 [1] and formalized in 1989 [2, 3].
- Its idea is simple; it keeps a **record of the previously searched candidate solutions** so that the algorithm does not check them again redundantly.
- In other words, those already searched candidate solutions are **taboo (tabu)** to be searched again.



Tabu List

$$\frac{1}{t^2+1} \rightarrow O\left(\frac{1}{t^2}\right) \quad O\left(\frac{1}{t}\right) \quad O(1)$$



- We have a tabu list which is a memory of previous candidate solutions which we have searched for.
- Tabu search uses space/memory as a trade-off with not checking redundant solutions.
- This list has a maximum length not to spend too much memory. If the length of the tabu list is ℓ , the tabu list contains the last ℓ candidate solutions and does not repeat them.
- We can have several tabu lists, with different lengths, named short-term memory, medium-term memory, and long-term memory. These memories have different lengths where the list of the long-term memory has more length.
- Based on the qualities of candidate solutions and/or pre-defined rules, we can put any visited candidate solution in one or several of these lists. The rules are defined by the user and the problem. For example, if a candidate solution contains some specific values for some specific features, we put them in the medium-term memory.

$$P \subseteq NP$$

$$O(t) \quad O(t^2) \quad O(t^3 + t^2) = O(t^3)$$

$$\Omega(t^3 + t^2) = \Omega(t^2)$$

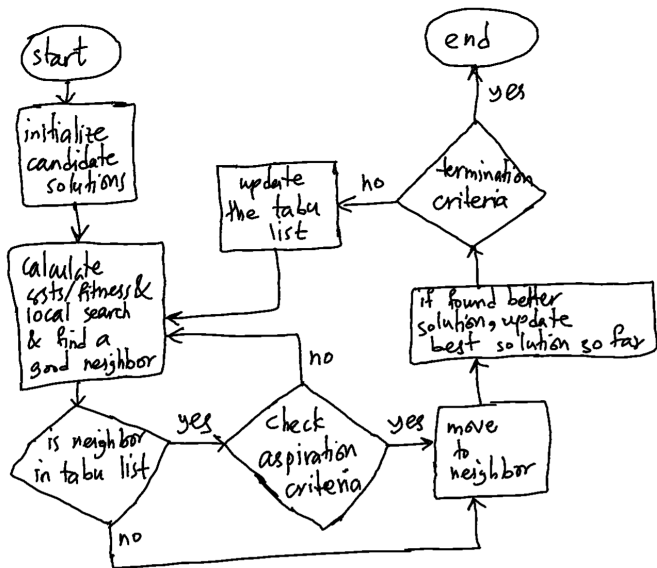


Aspiration Criteria

- When using tabu search, we can have aspiration criteria which are rules which allow a tabu move (searching an already checked candidate solution) to be accepted if it satisfies some condition.
- For example, a common aspiration criterion is to accept a tabu move if its cost function is the best solution found so far.
- In other words, during optimization, if we do not find good solutions for a while, it means that we have got stuck in some bad regions of the optimization landscape. In these cases, we can go and continue searching from one of the tabu candidate solution which had the best solution so far. From that point, we continue the search but in another direction.



Tabu Search Flowchart



Tabu Search Algorithm

Algorithm Tabu Search

Initialize the candidate solutions $\{x_1, \dots, x_n\}$ and tabu list $\mathcal{T} \leftarrow \emptyset$

while not converged do

$x^\dagger \leftarrow$ Perform metaheuristic optimization such as local search

if $x^\dagger \in \mathcal{T}$ **then**

if aspiration criteria are satisfied **then**

$x \leftarrow x^\dagger$

else

continue the loop (go to next loop)

else

$x \leftarrow x^\dagger$

if $f(x) <$ best cost so far **then**

best cost so far $\leftarrow f(x)$

best solution so far $\leftarrow x$

if termination criteria are satisfied **then**

Terminate the loop

else

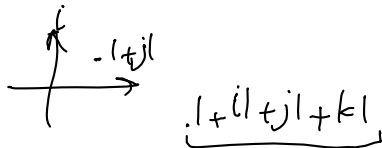
$\mathcal{T} \leftarrow \mathcal{T} \cup x$

Return the best solution so far

Acknowledgment

- Some slides of this slide deck are inspired by teachings of Prof. Saeed Sharifian at the Amirkabir University of Technology, Department of Electrical Engineering.
- Some papers and books about Tabu search: [4, 5, 6]

References



A handwritten diagram in the complex plane. A horizontal axis and a vertical axis intersect at the origin. A vector starts at the origin and points into the first quadrant. The vector is labeled with $-1+j$ near its tip. Below the vector, the expression $\sqrt{1+1+1}$ is written and underlined.

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