ECE 457A TUTORIAL 07: GAME THEORY

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Strategies To Solve Examples

- Dominant Strategy
  - It yields the best payoff for the player, regardless of other players' strategies.
- Iterated-Dominance (Dominated) Strategy
  - There is another strategy that performs at least as good, regardless of other players' strategies.
- Nash Equilibrium
  - A strategy profile is NE if no player wants to unilaterally deviate to another strategy, given other players' strategies.
  - No player can gain by deviating alone, i.e., by changing his or her strategy single-handedly.

Example 1 (Prisoner's Dilemma) - DS

		Prisoner 2: Confess	Prisoner 2: Deny
Prisoner 1	Confess	-10, -10	-1, -25
	Deny	-25, -1	-3, -3

DS for P1 → { Confess: -10-1 = -11 ✓ ; Deny: -25-3 = -28 X }

DS for P2 → { Confess: -10-1 = -11 ✓ ; Deny: -25-3 = -28 X }

(Confess, Confess) → dominant strategy equilibrium

Example 1 (Prisoner's Dilemma) - IDS

For P2, Deny (-25) is completely dominated by Confess (-10).

		Prisoner 2	
		Confess	Deny
Prisoner 1	Confess	-10, -10	-1, -25
	Deny	-25, -1	-3, -3

For P1, confess (-10) dominates Deny (-25)

(-10, -10) ⟶ (confess, confess) ⟶ IDS Eq

Example 1 (Prisoner's Dilemma) - NE

		Prisoner 2: Confess	Prisoner 2: Deny
Prisoner 1	Confess	-10, -10	-1, -25
	Deny	-25, -1	-3, -3

NE (confess, confess)

Example 2 - DS

DS for P1 → { Run : 1 − 1 = 0
Stop : 2 + 2 = 4 ✓

		Alice P2	
		Run	Stop
Bob P1	Run	1, 1	-1, -3
	Stop	2, -1	2, 3

DS for P2 → { Run : 1 − 1 = 0
Stop : −3 + 3 = 0
⟹ has no DS

Example 2 - IDS

Considering P1 ⟶

Stop dominates Run

$\begin{cases} 2 > 1 \\ 2 > -1 \end{cases}$

		Alice P2	
		Run	Stop
Bob P1	Run	1, 1	-1, -3
	Stop	2, -1	2, 3

Considering P2 ⟶ Stop dominates Run $(3 > -1)$

IDS Eq : (Stop, Stop)

Example 2 - NE

		Alice	
		Run	Stop
Bob	Run	1, 1	-1, -3
	Stop	2, -1	2, 3

P1 P2

2 > -1

3 > -1

NE (stop, stop)

Example 3 - DS

P1 → { A: 5+4 = 9 ; B: 3+3 = 6 X ; C: 2+4 = 6 X ; D: 4+5 = 9 } → we cannot find a dominant strategy

Player 1 \ Player 2	X	Y
A	5,2	4,2
B	3,1	3,2
C	2,1	4,1
D	4,3	5,4

P2 → { X: 2+1+1+3 = 7 ; Y: 2+2+1+4 = 9 ✓ → DS }

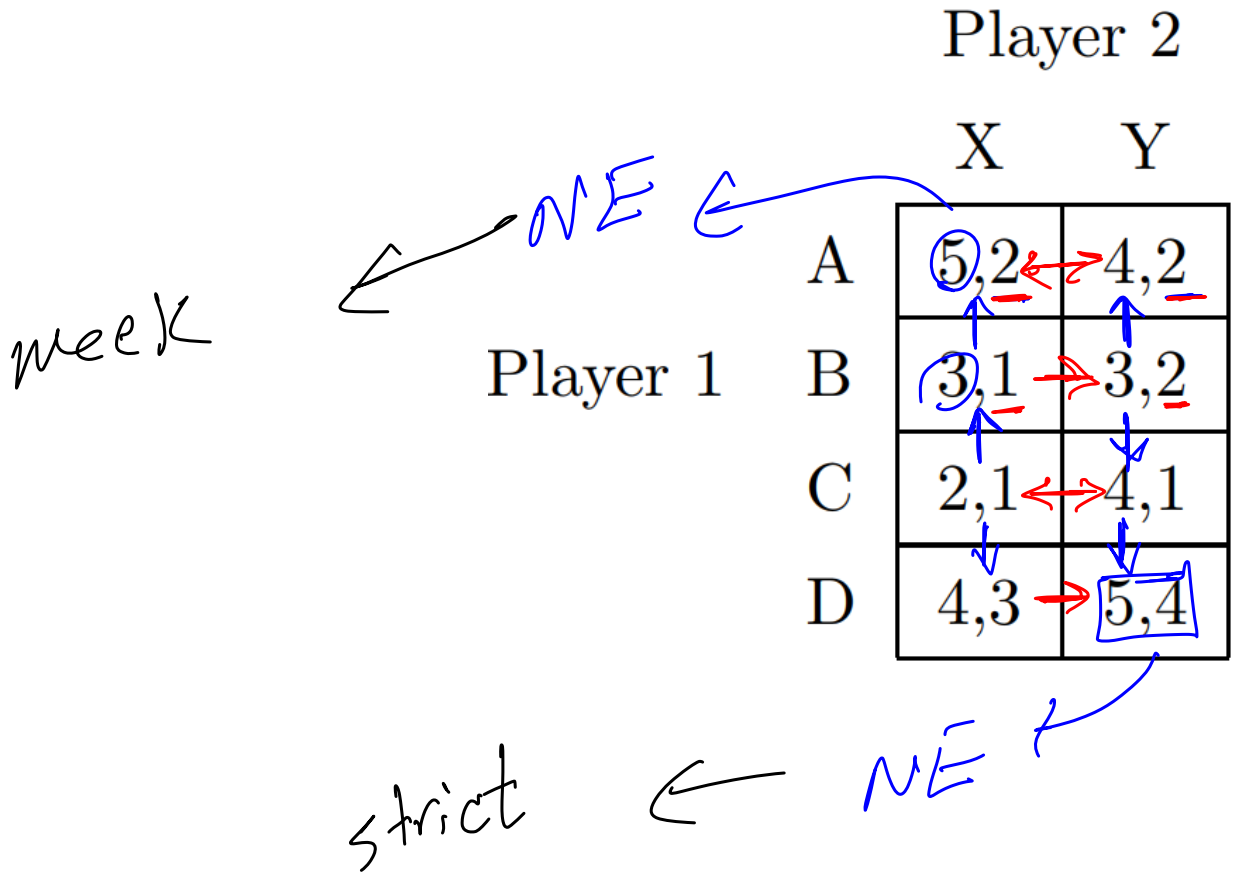
Example 3 - IDS

		Player 2 X	Player 2 Y
Player 1	A	5,2	4,2
	B	3,1	3,2
	C	2,1	4,1
	D	4,3	5,4

(D, Y)

5 > 4 = 4 > 3

Example 3 - NE


Player 2
X
Y
A
B
C
D
Player 1
5,2
4,2
3,1
3,2
2,1
4,1
4,3
5,4
NE
weak
strict
NE

Example 4 - DS

DS for P1 → { U: 44, M: 66 ✓, D: 16 }

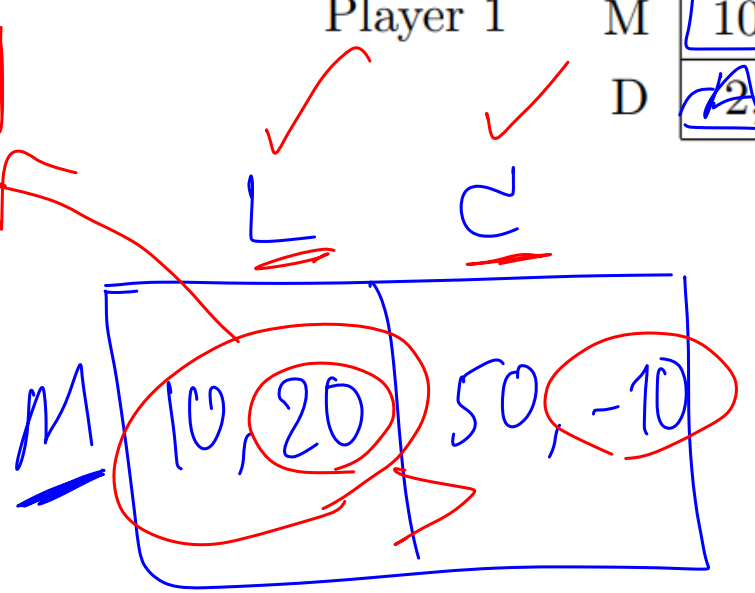
		Player 2		
		L	C	R
Player 1	U	1, 10	3, 20	40, 0
	M	10, 20	50, -10	6, 0
	D	2, 20	4, 40	10, 0

DS for P2 → { L: 50, C: 50, R: 0 } → P2 has no DS

Example 4 - IDS

Player 1 \ Player 2	L	C	R
U	1, 10	3, 20	40, 0
M	10, 20	50, -10	6, 0
D	2, 20	4, 40	10, 0

IDS Eq: (M, L)



Example 4 - NE

		Player 2		
		L	C	R
Player 1	U	1, 10	3, 20	40, 0
	M	10, 20	50, -10	6, 0
	D	2, 20	4, 40	10, 0

(M, L)

NE

References

- https://www.tayfunsonmez.net/wp-content/uploads/2013/10/E308SL4.pdf
- http://www.smallparty.com/yoram/classes/principles/nash.pdf