ECE 457A TUTORIAL 09: GAME THEORY

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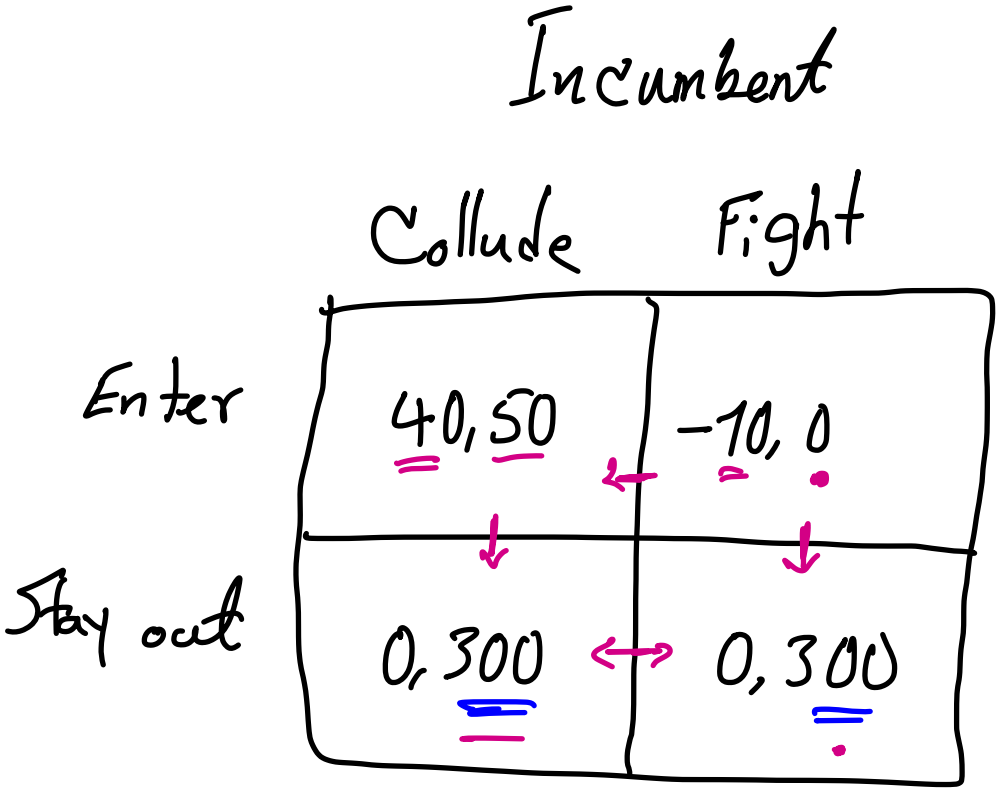
Dynamic Games (Entry Deterrence)

① players → Two firms

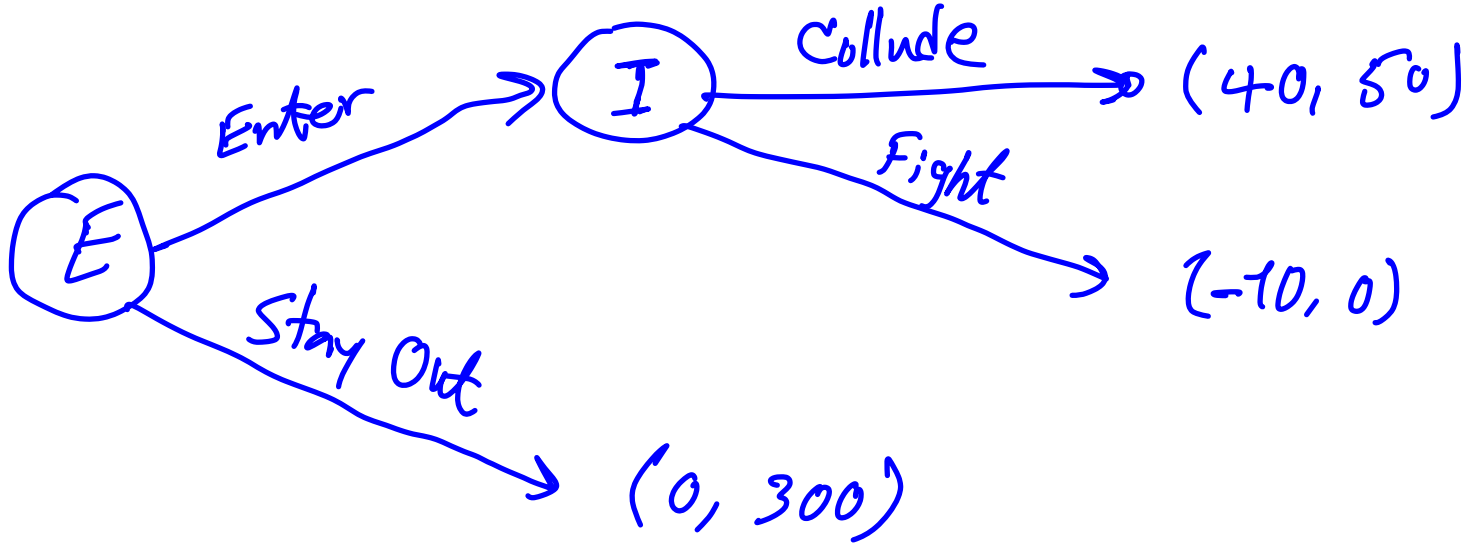
② The order of play

(1) → The Entrant decides whether to enter or stay out

(2) → If the Entrant enters, the Incumbent can collude or fight by cutting the price drastically



② Payoffs ⟶ Market profits are 300 at the monopoly price and 0 at the fighting price



→ the Incumbent's best response

NE: (stay out, Fight)

payoffs: (E, I)

Infinitely Repeated Games

strategies

① Grim (Trigger)

* Start with "C"
* continue to choose "C" unless other player chooses "B" (or "A"); then, choose "B" (or "A")

P1 \ P2	A	B	C
A	5,5	0,0	12,0
B	0,0	2,2	0,0
C	0,12	0,0	10,10

* At every time slot, the players act simultaneously

② Tit-for-Tat

* start with "C"
* then repeatedly play the last strategy played by the opponent

③ Play "C" in odd moves and "A" in even moves

P1 → ① P2 → ②

P1 : C C C ...

P2 : C C C ...

P1 → ① P2 → ③

P1 : C C A A A A ...

P2 : C A C A C A ....

P1 → ③ P2 → ②

P1 : C A C A C A C A ---

P2 : C C A C A C A C ....

Minimax Theorem

strategy x : maximum st

strategy y : minimum st

① $\min_{\underline{y}} \max_{\underline{x}} f(x,y) = -3x^2 + y^2 - 4xy$

② $\max_{x} \min_{\underline{y}} f(x,y) = -3x^2 + y^2 - 4xy$

① $\frac{\partial f}{\partial x} = 0 \Rightarrow -6x - 4y = 0 \Rightarrow x = \frac{-2}{3}y$ (*)

$f(y) = 3\left(\frac{-2}{3}y\right)^2 + y^2 - 4\left(\frac{-2}{3}y\right)y = 5y^2$

$\frac{4}{3}$ $\quad$ $\frac{8}{3}$

$\frac{\partial f(y)}{\partial y} = 10y = 0 \Rightarrow \boxed{y = 0 \Rightarrow x = 0}$

$\begin{cases} x^* = 0 \\ y^* = 0 \end{cases}$

② $\frac{\partial f}{\partial y} = 2y - 4x = 0 \Rightarrow y = 2x$ (*)

$f(x) = -3x^2 + \underbrace{(2x)^2}_{4} - \underbrace{4x(2x)}_{-8} = -7x^2$

$\frac{\partial f(x)}{\partial x} = -14x = 0 \Rightarrow x = 0 \Rightarrow y = 0 \Rightarrow \begin{cases} x^* = 0 \\ y^* = 0 \end{cases}$

$$\min_y \max_x f(x,y) = \max_x \min_y f(x,y)$$