# ECE 457A - Tutorial 4 - Example

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# Example: Supply and Demand for Vehicles

- A dealership sells two types of vehicles, trucks and sedans, and needs to choose the prices that maximize the revenue.
- To solve the problem, a mathematical model of sales is needed.
- Let us denote
  - by $x$ the price of a truck;
  - by $y$ the price of a sedan; and
  - by $t$ and $s$ the demands per month on trucks and sedans.

## Price-Demand Equations

$$t = 10,000 - 2x + 2.5y \quad (1)$$

$$s = 16,000 + 1.5x - 3y \quad (2)$$

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# Example: Supply and Demand for Vehicles

- Assume that the dealership sells
  - $t$ trucks at a price of $x$ dollars per truck and
  - $s$ sedans at a price of $y$ dollars per sedan.

## Revenue Equation

$$R = tx + sy = (10,000 - 2x + 2.5y)x + (16,000 + 1.5x - 3y)y \tag{3}$$

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# Example: Supply and Demand for Vehicles

- **Optimization Problem:**
  - **Find values of x and y such that R(x, y) achieves its maximal value.**

**Revenue Equation**

$$R(x, y) = -2x^2 - 3y^2 + 4xy + 10,000x + 16,000y \tag{4}$$

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# Example: Supply and Demand for Vehicles

- Maximizing $R$ is equal to minimizing $-R$.
- Therefore, based on first-order optimality condition, if $\mathbf{x}^*$ is a local minimizer for a differentiable function $f(\cdot)$, then $\nabla f(\mathbf{x}^*) = 0$.
- Here $f(\cdot) = -R(x, y)$ and $\mathbf{x}^* = [x \quad y]^T$.
- So

## System of Equations

$$\frac{\partial f}{\partial x} = -4x + 4y + 10,000 = 0 \tag{5}$$

$$\frac{\partial f}{\partial y} = -6y + 4x + 16,000 = 0 \tag{6}$$

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- Solving the system of equations, we obtain:
  - $x = 15,500$ for a truck;
  - $y = 13,000$ for a sedan; and
  - $R(x, y) = 181,500,000$.

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# References

Matthew F. Causley, Ruben Hayrepetyan, Allan Taylor (2017)
Multivariable Calculus Module II: Optimization
*Kettering University*, 1–14.